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Massive Rigid String Model and its Supersymmetric Extension

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Abstract

We discuss a rigid string model proposed by Casalbuoni and Longhi. Constraints for the massive states are solved to find the physical states and the mass spectrum. We also find its supersymmetric extension with the kappa symmetry. The supersymmetry transformations are found starting from on-shell transformations using the Dirac bracket.

1 Introduction

The superstring theories are promising approaches to the unified theory of fundamental interactions. The original bosonic string action is given in a geometrical form of the Nambu-Goto action [1, 2]. A possible modification was considered by adding an extrinsic curvature term[3]. It is sometimes called the rigid string since the additional term introduces rigidity of string [4, 5, 6].

The "rigid string" model we will discuss in this paper is one originally proposed by Casalbuoni and Longhi[7]. It is a bi-local type model and the action is proportional to the area of the world sheet swept out by the rigid string. In contrast to the rigid string of [3] string rigidity is built in the action geometrically. It is described by two end point coordinates $x_j^\mu(\tau), (j = 1, 2)$ and the string stretches straight in the relative coordinate direction $r^\mu = x_1^\mu - x_2^\mu$ then the action can be written as[1]

$$S = -T \int \left(\frac{1}{2}\sqrt{|dx_1^{[\mu} r^{\nu]}|^2} + \frac{1}{2}\sqrt{|dx_2^{[\mu} r^{\nu]}|^2} \right) = \int L \, d\tau, \tag{1.1}$$

where $dx_j^{[\mu} r^{\nu]}$ is the surface element spanned by dx_j^μ and r^ν. Then we have a NG type Lagrangian

$$L \;\; = \;\; -\frac{T}{2}\left(\sqrt{(\dot{x}_1 r)^2 - r^2(\dot{x}_1)^2} + \sqrt{(\dot{x}_2 r)^2 - r^2(\dot{x}_2)^2}\right). \tag{1.2}$$

It is important that direction of the relative coordinate $r^\mu = x_1^\mu - x_2^\mu$ is dynamically specified through the constraints from the action.

In [7] the massless sectors of the model was examined in detail and it was shown that the model describes massless gauge particles, photon, gravitons, and so on, with appropriate polarization properties. The action (1.2) also allow "massive string states" whose classical motion is corresponding to a rotating rod. The mass square of the string is proportional to the angular momentum.

In the first part of this paper we discuss the massive sector of the model in the hamiltonian formalism. There appear the second class constraints specifying that the relative coordinate and momentum are orthogonal to the total momentum. Thus the internal motion is described by the transverse coordinates and momenta satisfying further constraints from the lagrangian (1.2). The internal symmetries are SU(1,1) as well as the SO(d-2) and the physical states are constructed and their mass spectrum is determined.

In the second part we consider a supersymmetric extension of the model.[2] The NG type Lagrangian (1.2) can have space-time supersymmetry by introducing target space spinors. As usual the local fermionic invariance, the kappa symmetry, requires additional WZ type lagrangian. Although it is constructed cohomologically[11, 12] in the superstring theories we cannot apply it since the space of the coordinates are two world lines rather than two dimensional world sheet. However we can construct a kappa invariant action in a similar form as the D0 particle WZ action[13].

[1]There is a similar approach known as "straight string model" having different lagrangian [8, 9].

[2]The supersymmetic string action with extensic curvature has beed developed in [10]

The organization of the paper is as follows. In the next section we make the hamiltonian analysis of the lagrangian (1.2) and show the massive and massless sectors as possible branches of the system. In sect.III we quantize the bosonic model to find the physical states. In sect.IV we generalize it to the supersymmetric model and find possible kappa symmetric extension. In sect.V we analyze it in the hamiltonian formalism to find the supersymmetry generators. The last section is devoted to summary and discussions. In Appendix we give a form of off-shell supersymmetry transformations.

2 Bosonic Rigid String; Constraints and Hamiltonian

The bosonic lagrangian of the rigid string (1.2) is written as

$$L \;=\; -\frac{T}{2}\sqrt{r^2}\left(\sqrt{-(v_{1\perp})^2}+\sqrt{-(v_{2\perp})^2}\right), \tag{2.1}$$

where

$$v_i^\mu=\dot{x}_i^\mu, \qquad v_{i\perp}^\mu=(\eta^{\mu\nu}-\frac{r^\mu r^\nu}{r^2})v_{i\nu}, \qquad r^\mu=x_1^\mu-x_2^\mu, \qquad (i=1,2) \tag{2.2}$$

and T is a constant[3] with dimension $[T]=[\ell^{-2}]$. It is a singular lagrangian and examined by using the generalized hamiltonian formalism[15]. The momenta conjugate to the coordinates $x_i^\mu,(i=1,2)$ are

$$p_{i\mu}=\frac{\frac{T}{2}\sqrt{r^2}}{\sqrt{-(v_{i\perp})^2}}\;v_{i\perp\mu}. \tag{2.3}$$

They are not independent but satisfy following four primary constraints

$$\phi_i=\frac{1}{2}\left(p_i^2+(\frac{T}{2})^2r^2\right)=0, \qquad \psi_i=p_ir=0. \tag{2.4}$$

The generalized Hamiltonian $p_i\dot{x}_i-L$ is[16, 17]

$$H=\frac{\sqrt{-(v_{i\perp})^2}}{T\sqrt{r^2}}\;\left(p_i^2+(\frac{T}{2})^2r^2\right)+\frac{v_ir}{r^2}\;p_ir-\frac{\sqrt{-(v_{i\perp})^2}}{T\sqrt{r^2}}\left(p_{i\mu}-\frac{\frac{T}{2}\sqrt{r^2}}{\sqrt{-(v_{i\perp})^2}}v_{i\perp\mu}\right)^2. \tag{2.5}$$

The last term is "squire of the definition of momenta" (2.3) and does not contribute in the equations of motion. The hamiltonian is expressed as a sum of the four primary constraints,

$$H \;=\; \lambda_j\;\phi_j+\mu_j\;\psi_j=\lambda_j\;\frac{1}{2}\left(p_j^2+(\frac{T}{2})^2r^2\right)+\mu_j\;(p_jr), \tag{2.6}$$

where the multipliers λ_j and μ_j are given in terms of undetermined velocities as in (2.5),

$$\lambda_j=\frac{\sqrt{-(v_{j\perp})^2}}{\frac{T}{2}\sqrt{r^2}}, \qquad \mu_j=\frac{v_jr}{r^2}. \tag{2.7}$$

[3]Models with non-constant $T=T(r^2)$ are discussed in [14].

Note both λ_1 and λ_2 are positive in order that the Euler-Lagrange equations are reproduced from the Hamilton equations.

The consistency condition that the primary constraints are conserved in time gives

$$\partial_\tau \phi_i = \begin{pmatrix} 2(\frac{T}{2})^2 r^2 & -p_1 p_2 - (\frac{T}{2})^2 r^2 \\ p_1 p_2 + (\frac{T}{2})^2 r^2 & -2(\frac{T}{2})^2 r^2 \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = 0, \tag{2.8}$$

$$\partial_\tau \psi_i = \begin{pmatrix} -2(\frac{T}{2})^2 r^2 & -p_1 p_2 - (\frac{T}{2})^2 r^2 \\ p_1 p_2 + (\frac{T}{2})^2 r^2 & 2(\frac{T}{2})^2 r^2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = 0. \tag{2.9}$$

To have non trivial motion it is necessary that the determinant of the above matrices vanish. It is satisfied in the following two cases

$$p_1 p_2 + (\frac{T}{2})^2 r^2 = \pm 2(\frac{T}{2})^2 r^2. \tag{2.10}$$

The upper sign solution of (2.10) gives

$$p_1 p_2 - (\frac{T}{2})^2 r^2 = 0, \quad \mu_1 = \mu_2, \quad \lambda_1 = -\lambda_2. \tag{2.11}$$

Since λ_1 and λ_2 have opposite signs this case is discarded. The lower sign solution of (2.10) gives

$$\chi \equiv p_1 p_2 + 3(\frac{T}{2})^2 r^2 = 0, \quad \mu_1 = -\mu_2, \quad \lambda_1 = \lambda_2 \equiv \lambda. \tag{2.12}$$

The $\chi = 0$ is the secondary constraint and it further requires

$$\partial_\tau \chi = (\mu_1 - \mu_2)\, T^2\, r^2 = 2\, \mu_1\, T^2\, r^2 = 0. \tag{2.13}$$

It gives either $\mu_1 = 0$ or $r^2 = 0$. The former case gives

$$\mu_1 = -\mu_2 = 0 \tag{2.14}$$

and no more constraint appears. It gives massive states as we will study in below. In the latter case $r^2 = 0$ is the tertiary constraint and the set of constraints is

$$p_1^2 = p_2^2 = p_1 p_2 = p_1 r = p_2 r = r^2 = 0. \tag{2.15}$$

They are first class constraints and conserved for any λ_i, μ_i. The constraints (2.15) mean the massless states $P^2 = (p_1 + p_2)^2 = 0$. Thus the lagrangian system is describing both massless and massive sectors. The massless sectors are examined in detain in [7] and it was shown that massless gauge particle states appear in the quantized spectrum. In the following we will discuss the massive sector.

We consider the case of (2.12) and (2.14) then the hamiltonian becomes

$$H = \lambda(\phi_1 + \phi_2) = \frac{\lambda}{2}(p_1^2 + p_2^2 + \frac{T^2}{2} r^2) \equiv \lambda\, \phi. \tag{2.16}$$

Introducing center of mass and relative coordinates as

$$P_\mu = p_{1\mu} + p_{2\mu}, \quad X^\mu = \frac{1}{2}(x_1^\mu + x_2^\mu), \quad q_\mu = \frac{1}{2}(p_{1\mu} - p_{2\mu}), \quad r^\mu = x_1^\mu - x_2^\mu, \tag{2.17}$$

the primary constraints (2.4) and secondary constraints (2.10) are

$$Pr = Pq = qr = q^2 - (\frac{T}{2})^2 r^2 = 0, \quad \text{and} \quad \phi = \frac{1}{4}P^2 + (q^2 + (\frac{T}{2})^2 r^2) = 0. \tag{2.18}$$

$\phi = 0$ is the first class constraint and appearing in the Hamiltonian with arbitrary multiplier $\lambda(\tau)$. Other 4 constraints are the second class. We first eliminate $Pr = Pq = 0$ at classical stage using a canonical transformation[18, 19]. We also consider the bosonic system in 4 dimensions so that the symmetry algebra in the transverse space is SO(3). It leaves transverse relative three coordinates $\mathbf{u}$ and relative three momenta $\mathbf{v}$ subject to the constraints

$$\mathcal{T}_1 \equiv \frac{1}{2T}(\mathbf{v}^2 - (\frac{T}{2})^2 \mathbf{u}^2) = 0, \quad \mathcal{T}_2 \equiv \frac{1}{2}\mathbf{v}\mathbf{u} = 0 \tag{2.19}$$

and the first class constraint

$$\phi = \frac{1}{4}P^2 + 2T\,\mathcal{T}_0 = 0, \qquad \mathcal{T}_0 \equiv \frac{1}{2T}(\mathbf{v}^2 + (\frac{T}{2})^2 \mathbf{u}^2). \tag{2.20}$$

The last one fixes the mass of the system as

$$M^2 = -P^2 = 8\,T\,\mathcal{T}_0. \tag{2.21}$$

Classically the mass is determined in terms of the angular momentum

$$\mathcal{L} = \mathbf{u} \times \mathbf{v} \tag{2.22}$$

as follows. Using constraints $\mathcal{T}_1 = \mathcal{T}_2 = 0$ in (2.19),

$$\mathcal{L}^2 = (\mathbf{u} \times \mathbf{v})^2 = (\mathbf{u})^2(\mathbf{v})^2 - (\mathbf{u}\mathbf{v})^2 = (\frac{T}{2})^2(\mathbf{u}^2)^2. \tag{2.23}$$

Then the $(\text{mass})^2$ is proportional to the length of the angular momentum,

$$M^2 = -P^2 = 2T^2(\mathbf{u}^2) = 4T\,|\mathcal{L}|. \tag{2.24}$$

3 Quantization and Physical States

We discuss the quantization of the transverse variables $(\mathbf{u}, \mathbf{v})$ subject to the constraints (2.19). The $\mathcal{T}_0$, therefore the Hamiltonian, is diagonalized using ladder operators

$$a_r = \frac{1}{\sqrt{T}}(v_r - i\frac{T}{2}u_r), \qquad a_r^\dagger = \frac{1}{\sqrt{T}}(v_r + i\frac{T}{2}u_r), \qquad [a_r, a_s^\dagger] = \delta_{rs}. \tag{3.1}$$

The operators $\mathcal{T}_1$ and $\mathcal{T}_2$ in (2.19) and $\mathcal{T}_0$ in (2.20) are

$$\mathcal{T}_- = \mathcal{T}_1 - i\,\mathcal{T}_2 = \frac{1}{2}a_r^2, \qquad \mathcal{T}_+ = \mathcal{T}_1 + i\,\mathcal{T}_2 = \frac{1}{2}(a_r^\dagger)^2, \qquad \mathcal{T}_0 = \frac{1}{2}(a_r^\dagger a_r + \frac{3}{2}) \tag{3.2}$$

and form the SU(1,1) algebra

$$[\mathcal{T}_0, \mathcal{T}_\pm] = \pm\mathcal{T}_\pm, \qquad [\mathcal{T}_+, \mathcal{T}_-] = 2\mathcal{T}_0. \tag{3.3}$$

The first class constraint $\phi = 0$ in (2.20) is hermitic and is imposed as the physical state condition

$$\phi\,|phys\rangle = \left(\frac{1}{4}P^2 + 2T\,\mathcal{T}_0 + c_0\right)|phys\rangle = 0, \tag{3.4}$$

where the constant c_0 is an ordering ambiguity. Therefore the mass of the physical state $|phys\rangle$ is determined as

$$M^2\,|phys\rangle = -P^2\,|phys\rangle = 4\,(2T\,\mathcal{T}_0 + c_0)\,|phys\rangle. \tag{3.5}$$

The second class constraints $\mathcal{T}_1 = \mathcal{T}_2 = 0$ in (2.19) at quantum theory are imposed as a physical state condition *a.la.* Gupta-Bleuler,

$$\mathcal{T}_-\,|phys\rangle = \frac{1}{2}\,a_r^2\,|phys\rangle = 0. \tag{3.6}$$

We can solve them to find the physical states and their mass spectrum.

In order to find the physical states we consider eigenstates of the angular momentum $\mathcal{L}$ in (2.22),

$$\mathcal{L}_r = -i\,\epsilon_{rst}\,a_s^\dagger a_t. \tag{3.7}$$

The internal rotations SO(3) is a symmetry of the model and the generators $\mathcal{L}_r$ commute with those of SU(1,1), $\mathcal{T}_\pm$ and $\mathcal{T}_0$. $\mathcal{L}_3 = -i(a_1^\dagger a_2 - a_2^\dagger a_1)$ is not diagonal in a_r but can be diagonalized by a unitary transformation

$$\mathbf{a} = U\mathbf{b}, \qquad \mathbf{a}^\dagger = \mathbf{b}^\dagger U^\dagger, \qquad U = \begin{pmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ -i/\sqrt{2} & 0 & -i/\sqrt{2} \\ 0 & 1 & 0 \end{pmatrix}. \tag{3.8}$$

In terms of b_r the SO(3) generators (3.7) are

$$\begin{aligned} \mathcal{L}_3 &= b_1^\dagger b_1 - b_3^\dagger b_3, \\ \mathcal{L}_+ &= \mathcal{L}_1 + i\mathcal{L}_2 = \sqrt{2}\,(\,b_1^\dagger b_2 + \,b_2^\dagger b_3\,), \\ \mathcal{L}_- &= \mathcal{L}_1 - i\mathcal{L}_2 = \sqrt{2}\,(\,b_2^\dagger b_1 + \,b_3^\dagger b_2\,). \end{aligned} \tag{3.9}$$

The SU(1,1) generators in terms of b_r are

$$\begin{aligned} \mathcal{T}_0 &= \frac{1}{2}(b_1^\dagger b_1 + b_2^\dagger b_2 + b_3^\dagger b_3 + \frac{3}{2}), \\ \mathcal{T}_+ &= \mathcal{T}_1 + i\mathcal{T}_2 = \frac{1}{2}({b_2^\dagger}^2 - 2\,b_1^\dagger b_3^\dagger\,), \\ \mathcal{T}_- &= \mathcal{T}_1 - i\mathcal{T}_2 = \frac{1}{2}({b_2}^2 - 2\,b_1 b_3)\,. \end{aligned} \tag{3.10}$$

General Fock states are constructed by applying $b_r^\dagger$'s on the ground state $|0\rangle$,

$$\frac{1}{\sqrt{j_1!j_2!j_3!}}(b_1^\dagger)^{j_1}(b_2^\dagger)^{j_2}(b_3^\dagger)^{j_3}|0\rangle, \qquad b_r|0\rangle = 0. \tag{3.11}$$

They are the eigenstates of $\mathcal{T}_0$ with eigenvalue $\frac{1}{2}(j_1+j_2+j_3+\frac{3}{2}) \equiv \frac{1}{2}(j+\frac{3}{2})$. The number of such states for a fixed value of j is

$$\sum_{j_1=0}^{j}\sum_{j_2=0}^{j-j_1} 1 = \frac{(j+1)(j+2)}{2}. \tag{3.12}$$

They are decomposed into irreducible representations of SO(3),

$$\frac{(j+1)(j+2)}{2} = \sum_{n=j,j-2,j-4,..}(2n+1). \tag{3.13}$$

That is sum of spin $j, j-2, j-4, ...$ multiplets. Among them the highest spin j multiplet is constructed from

$$|j,-j\rangle = \frac{1}{\sqrt{j!}}(b_3^\dagger)^j|0\rangle, \tag{3.14}$$

by multiplying $\mathcal{L}_+$ successively. Here $|j,m\rangle$ is the eigenstate of $\mathcal{L}^2$ and $\mathcal{L}_3$ with eigenvalues $j(j+1)$ and $m, (|m| \leq j)$. They are satisfying the physical state condition (3.6) since $\mathcal{T}_-|j,-j\rangle = 0$ and $[\mathcal{T}_-,\mathcal{L}_+] = 0$. Other low lying states with spin $j-2, j-4, ...$ are unphysical states and are given by

$$|j-2r,m\rangle = N\ (\mathcal{T}_+)^r(\mathcal{L}_+)^{m+j-2r}(b_3^\dagger)^{j-2r}|0\rangle, \quad r=1,2,..,[\frac{j}{2}],\ |m| \leq j-2r. \tag{3.15}$$

with a normalization factor N.

In summary the physical states are

$$|j,m\rangle = \sqrt{\frac{(j-m)!}{(2j)!(j+m)!}}\,(\mathcal{L}_+)^{m+j}\frac{1}{\sqrt{j!}}(b_3^\dagger)^j|0\rangle, \qquad 0 \leq j,\ \ -j \leq m \leq j, \tag{3.16}$$

and the mass of the states is

$$M^2|j,m\rangle = 4(2T\ \mathcal{T}_0 + c_0)\ |j,m\rangle = (4T\ (j+\frac{3}{2}) + 4c_0)\ |j,m\rangle. \tag{3.17}$$

where c_0 coming from the ordering ambiguity is not determined, for example from the Lorentz invariance. The spectrum (3.17) is corresponding to the classical one in (2.24) that the mass is coming from the internal rotation energy. They have maximal spin lying on the leading Regge trajectory and their motion is corresponding to rotating rod classically.

In the above we have eliminated two of the second class constraints $Pr = Pq = 0$ classically. Alternatively we could impose them in quantum theory using covariant

oscillators $a_\mu = \frac{1}{\sqrt{T}}(q_\mu - i\frac{T}{2}r_\mu)$. In terms of them five constraints in (2.18) are

$$\begin{aligned} L_{-2} &\equiv \frac{1}{2}(a_\mu^\dagger)^2, \qquad L_{-1} \equiv \frac{1}{\sqrt{2T}}\, Pa^\dagger, \qquad L_0 \equiv \frac{P^2}{4T} + (a_\mu^\dagger a^\mu) \\ L_2 &\equiv \frac{1}{2}(a_\mu)^2, \qquad L_1 \equiv \frac{1}{\sqrt{2T}}\, Pa, \end{aligned} \tag{3.18}$$

They are obtained from the Virasoro generators of the NG string by truncating the higher oscillator modes; $a_{n\mu} = 0, (n \geq 2)$. In contrast to the Virasoro generators, which are the first class set classically, the constraints (3.18) are not first class. However we can impose them at quantum theory as the physical state conditions,

$$L_2|phys\rangle = L_1|phys\rangle = (L_0 - \alpha_0)|phys\rangle = 0. \tag{3.19}$$

They are solved as above by using the Lorentz transformation from the rest frame.

4 Supersymmetric Model

The Nambu-Goto action for the superstring is

$$L^{NG} = -\frac{T}{2}\left(\sqrt{(v_1 r)^2 - r^2(v_1)^2} + \sqrt{(v_2 r)^2 - r^2(v_2)^2}\right), \tag{4.1}$$

where v_i's are super invariant velocities. First we leave them in a general form as

$$v_i^\mu = \dot{x}_i^\mu + iB_i^{jk}\overline{\theta}_j\Gamma^\mu\dot{\theta}_k. \tag{4.2}$$

In this section we consider Majorana-Weyl spinors $\theta_j, (j = 1, 2)$ in 10 dimensions and $(\mathcal{C}\Gamma^A), (A = 0, 1, 2, ..., 9, 11)$ have symmetric gamma indices.[4] The relative coordinate is

$$r^\mu = x_1^\mu - x_2^\mu \tag{4.3}$$

since additional fermionic contributions, if any, can be absorbed into x_j^μ by redefinition.

The global susy transformation is determined from the susy invariance of v_i,

$$\delta_\epsilon\theta_i = \epsilon_i, \quad \delta_\epsilon x_i^\mu = -iB_i^{jk}\overline{\epsilon}_j\Gamma^\mu\theta_k, \qquad \rightarrow \quad \delta_\epsilon v_i^\mu = 0. \tag{4.4}$$

The susy transformation of r^μ is

$$\delta_\epsilon r^\mu \;=\; \delta(x_1^\mu - x_2^\mu) = -i(B_1^{jk} - B_2^{jk})\overline{\epsilon}_j\Gamma^\mu\theta_k. \tag{4.5}$$

For the invariance of r^μ it is sufficient to choose the coefficients B_i^{jk} as

$$B_1^{jk} = B_2^{jk} \equiv B^{jk}, \tag{4.6}$$

[4] We use the mostly positive metric $\eta_{\mu\nu} = (-; +...+)$ and the Clifford algebra is $\{\Gamma_\mu, \Gamma_\nu\} = 2\eta_{\mu\nu}$.

where B^{jk} is taken to be symmetric matrix since the anti-symmetric part can be absorbed into the definition of x_i in this case.

Kappa transformation is the local fermionic symmetry under which

$$\delta_\kappa \theta_j = \kappa_j(\tau), \qquad \delta_\kappa x_i^\mu = -iB^{jk}\overline{\theta}_j \Gamma^\mu \kappa_k(\tau), \tag{4.7}$$

then

$$\delta_\kappa v_i^\mu \;\; = \;\; -2i \; B^{jk} \; \dot{\overline{\theta}}_j \Gamma^\mu \kappa_k. \tag{4.8}$$

Under the choice (4.6) r^μ is kappa invariant also,

$$\delta_\kappa r^\mu \;\; = \;\; \delta(x_1^\mu - x_2^\mu) = 0. \tag{4.9}$$

The L^{NG} is invariant under the super transformation (4.4) with (4.6) while it transforms under the kappa transformation as

$$\delta_\kappa L^{NG} = p^i \delta v_i = -2i \; B^{jk} \; \dot{\overline{\theta}}_j \not{P} \kappa_k, \tag{4.10}$$

where $p_\mu^i = \frac{\partial L^{NG}}{\partial v_i^\mu}$. To compensate it we consider a possible additional lagrangian, corresponding to WZ one. Usually it appears from the discussions of non-trivial Chevalley-Eilenberg cohomology of the super Poincare group[11, 12]. In the case of superstring, there exists a closed susy invariant three form $\Omega^3 = d\Omega^2$. The Ω^2 is not an element of the Chevalley-Eilenberg cohomology and works as the WZ term. In the present case the world sheet is degenerated to the world lines at the boundaries due to the bi-local nature. However we can find the additional action in a form similar to the D0 particle WZ action[13] by replacing its mass to $r \equiv \sqrt{r^2}$,

$$L^{WZ} \;\; = \;\; i\, r\, C^{jk} \overline{\theta}_j \Gamma_{11} \dot{\theta}_k, \tag{4.11}$$

where C^{jk} is some constant matrix. It transforms under the susy

$$\delta_\epsilon L^{WZ} \;\; = \;\; i\, r\, C^{jk} \overline{\epsilon}_j \Gamma_{11} \dot{\theta}_k = \partial_\tau (i\, r\, C^{jk} \overline{\epsilon}_j \Gamma_{11} \theta_k) - i\, \dot{r}\, C^{jk} \overline{\epsilon}_j \Gamma_{11} \theta_k. \tag{4.12}$$

It is (pseudo) invariant if the supersymmetry parameters ϵ_J is restricted by

$$\epsilon_j \; C^{jk} = 0. \tag{4.13}$$

However since $\dot{r} = 0$ is a result of the equation of motion, as we will see in (5.14),

$$\mu_1 - \mu_2 = \frac{(r\dot{r})}{r^2} = 0 \tag{4.14}$$

L^{WZ} is invariant for any supersymmetry parameters ϵ_J *on-shell* and the super charges will be introduced as in (5.20).[5]

[5]If a variation of a lagrangian $\delta L = (\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}})\delta q + \frac{d}{dt}(\frac{\partial L}{\partial \dot{q}}\delta q)$ is written non-trivially in a form $\frac{d}{dt}F + (eom)$, then $G = (\frac{\partial L}{\partial \dot{q}}\delta q - F)$ is a conserved quantity, where $(eom) = 0$ using equations of motion.

The L^{WZ} transforms under the kappa

$$\delta_\kappa L^{WZ} \;=\; -2ir\, C_+^{kj}\,\dot{\overline{\theta}}_j\Gamma_{11}\kappa_k - i\,\dot r\, C^{jk}\overline{\theta}_j\Gamma_{11}\kappa_k + \partial_\tau(i\,r\,C^{jk}\overline{\theta}_j\Gamma_{11}\kappa_k), \tag{4.15}$$

where $C_\pm^{jk} = \frac{1}{2}(C^{jk}\pm C^{kj})$ are symmetric and anti-symmetric parts of the constant matrix C^{jk}. The kappa variation of the total lagrangian is

$$\delta_\kappa L^{tot} \;=\; -2i\,\dot{\overline{\theta}}_j(B^{jk}\,\not{P} + C_+^{jk}\,r\,\Gamma_{11})\kappa_k - i\,\dot r\,C^{jk}\,\overline{\theta}_j\Gamma_{11}\kappa_k + \partial_\tau(ir\,C^{jk}\overline{\theta}_j\Gamma_{11}\kappa_k). \tag{4.16}$$

The action is kappa invariant if the kappa functions $\kappa_k(\tau)$'s satisfy

$$C^{jk}\,\kappa_k = 0 \tag{4.17}$$

and

$$\left(\delta^{jk}\,\not{P} + (B^{-1}C_+)^{jk}\,r\,\Gamma_{11}\right)\kappa_k = \left(\delta^{jk}\,\not{P} - (B^{-1}C_-)^{jk}\,r\,\Gamma_{11}\right)\kappa_k = 0. \tag{4.18}$$

In order to have non-trivial kappa transformations it is necessary to hold

$$[(B^{-1}C_-), C]_- \sim C. \tag{4.19}$$

There exists such matrices, for example

$$B \;=\; \frac{1}{\beta}\begin{pmatrix} 1-\beta & 1 \\ 1 & 1+\beta \end{pmatrix}, \qquad C = k\begin{pmatrix} \beta+1 & \beta+1 \\ \beta-1 & \beta-1 \end{pmatrix}, \tag{4.20}$$

$$C_- \;=\; k\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad (B^{-1}C_-)^2 = (\frac{k}{\beta})^2\begin{pmatrix} -1 & -(1+\beta) \\ (1-\beta) & 1 \end{pmatrix}^2 = k^2, \tag{4.21}$$

where k and β are non-zero real constants. Note if we take $k = \sqrt{2}\,T$,

$$\left(\not{P} - (B^{-1}C_-)\,r\,\Gamma_{11}\right)^2 = P^2 + 2\,T^2\,r^2 = 0 \tag{4.22}$$

using constraints from the lagrangian as we will see shortly. In this case we can write

$$\left(\not{P} - (B_+^{-1}C_-)\,r\,\Gamma_{11}\right) = \frac{1-\Gamma_\kappa}{2}\,2\not{P}, \qquad \Gamma_\kappa \equiv \sqrt{2}\,T\,r\,\Gamma_{11}\frac{1}{\not{P}}, \qquad \Gamma_\kappa^2 = 1 \tag{4.23}$$

and $\frac{1\pm\Gamma_\kappa}{2}$ work as the projection operators for the kappa transformations.

5 Hamiltonian Formalism of Supersymmetric Model

We are going to discuss the total Lagrangian (4.1)+(4.11),

$$L^{tot} = -\frac{T}{2}\left(\sqrt{(v_1 r)^2 - r^2(v_1)^2} + \sqrt{(v_2 r)^2 - r^2(v_2)^2}\right) + i\,r\,C^{jk}\overline{\theta}_j\Gamma_{11}\dot\theta_k. \tag{5.1}$$

with $v_i^\mu = \dot x_i^\mu + iB^{jk}\overline{\theta}_j\Gamma^\mu\dot\theta_k$. Since L^{WZ} does not depends on $\dot x_j$ the bosonic primary constraints appear in the same forms as in the bosonic model, (2.4),

$$\phi_i = \frac{1}{2}(p_i^2 + (\frac{T}{2})^2 r^2) = 0, \qquad \psi_i = p_i r = 0. \tag{5.2}$$

We define the fermionic momentum π^k conjugate to θ_k by the *right derivative*

$$\pi^k \;=\; \frac{\partial^r L^{tot}}{\partial \dot{\theta}_k} = iB^{jk}\overline{\theta}_j \not{P} + i\, r\, C^{jk}\overline{\theta}_j \Gamma_{11}, \tag{5.3}$$

then the fermionic primary constraints are

$$\zeta^k \;=\; \pi^k - i\overline{\theta}_j (B^{jk}\not{P} + C^{jk}\, r\, \Gamma_{11}) = 0. \tag{5.4}$$

The hamiltonian is expressed as a sum of the primary constraints,

$$H \;=\; \lambda_j\, \phi_j + \mu_j\, \psi_j + \zeta^k\, \rho_k. \tag{5.5}$$

where λ_j and μ_j are the bosonic multipliers given in the same forms as in (2.7) but with the v_i^μ in (4.2) and $\rho_k = \dot{\theta}_k$ is the fermionic multiplier. The consistency condition that the primary constraints are conserved in time gives

$$\partial_\tau \phi_i \;=\; \begin{pmatrix} 2(\frac{T}{2})^2 r^2 & -p_1p_2 - (\frac{T}{2})^2 r^2 \\ p_1p_2 + (\frac{T}{2})^2 r^2 & -2(\frac{T}{2})^2 r^2 \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = 0, \tag{5.6}$$

$$\partial_\tau \psi_i \;=\; \begin{pmatrix} -2(\frac{T}{2})^2 r^2 & -p_1p_2 - (\frac{T}{2})^2 r^2 \\ p_1p_2 + (\frac{T}{2})^2 r^2 & 2(\frac{T}{2})^2 r^2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} + i\, r\, C^{jk}\overline{\theta}_j \Gamma_{11}\rho_k \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0. \tag{5.7}$$

$$\partial_\tau \zeta^k \;=\; -2i\overline{\rho}_j (B_+^{jk}\not{P} + C_+^{jk}\, r\, \Gamma_{11}) - i\overline{\theta}_j C^{jk}\Gamma_{11}\, r\, (\mu^1 - \mu^2) = 0. \tag{5.8}$$

As in the bosonic case we get the secondary constraint from (5.6),

$$\chi \equiv p_1p_2 + 3(\frac{T}{2})^2 r^2 = 0, \qquad \mu_1 = -\mu_2. \tag{5.9}$$

Using it (5.7) requires

$$\lambda_1 = \lambda_2, \tag{5.10}$$

and

$$C^{jk}\rho_k = 0, \tag{5.11}$$

which for the choice of C^{jk} in (4.20) restricts ρ_k as

$$\rho_1 = -\rho_2 \equiv \rho. \tag{5.12}$$

The secondary constraint $\chi = 0$ in (5.9) further requires

$$\partial_\tau \chi = 2(\mu_1 - \mu_2)\, T^2\, r^2 = 4\, \mu_1^2\, r^2 = 0. \tag{5.13}$$

It gives, corresponding to the massive bosonic sector,

$$\mu_1 = -\mu_2 = 0 \tag{5.14}$$

and no more constraint appears. Finally (5.8) requires

$$(B_+^{jk}\not{P} + C_+^{jk}\, r\, \Gamma_{11})\rho_k = 0, \quad \rightarrow \quad (\not{P} - k\, r\, \Gamma_{11})\rho = 0. \tag{5.15}$$

The conditions (5.11) and (5.15) are corresponding to (4.17) and (4.18). To exist non-trivial ρ the $(\not{P} - k\,r\,\Gamma_{11})$ must be a projection operator. It determines $k = \sqrt{2}\,T$ as

$$(\not{P} - k\,r\,\Gamma_{11})^2 = P^2 + k^2\,r^2 = (p_1 + p_2)^2 + 2\,T^2\,r^2 = 0. \tag{5.16}$$

Then the Hamiltonian becomes

$$H \;=\; \lambda\,(\phi_1 + \phi_2) + (\zeta^1 - \zeta^2)\,(\not{P} - \sqrt{2}\,T\,r\,\Gamma_{11})\tilde{\rho}, \tag{5.17}$$

where $\tilde{\rho}$ is arbitrary MW spinor but only half components are independent due to the projector $(\not{P} - \sqrt{2}\,T\,r\,\Gamma_{11})$. The constraints appearing here belong to the first class.

$$\phi_1 + \phi_2 = \frac{1}{2}(p_1^2 + p_2^2 + \frac{T^2}{2}r^2) \equiv\; \phi \tag{5.18}$$

generates the τ reparametrization and determines the mass of the system.

$$\zeta \;\equiv\; (\zeta^1 - \zeta^2)\,(\not{P} - \sqrt{2}\,T\,r\,\Gamma_{11}) = \left\{\pi^1 - \pi^2 + i(\overline{\theta}_1 + \overline{\theta}_2)\not{P}\right\}(\not{P} - \sqrt{2}\,T\,r\,\Gamma_{11}) \tag{5.19}$$

generates the kappa symmetry.

We have seen the lagrangian is pseudo invariant only *on-shell* in (4.12). Although it is on-shell invariant we can introduce the global supercharges (see footnote 3),

$$Q^j = \pi^j + i\overline{\theta}_k(B^{jk}\not{P} + C^{jk}\;r\;\Gamma_{11}). \tag{5.20}$$

They are conserved and satisfying the super Poincare algebra, especially

$$\{Q^j, Q^k\}_+ = -2\left(B^{jk}(\mathcal{C}\not{P}) + C_+^{jk}\;r\;(\mathcal{C}\Gamma_{11})\right). \tag{5.21}$$

Here the last term is the central charge, in which $r = \sqrt{r^2}$ commutes with super Poincare generators, $(P_\mu, M_{\mu\nu}, Q^j)$.

There is one constraint $qr = (\psi_1 - \psi_2)/2 = 0$ having non-zero Poisson brackets with Q^j while other all constraints have weakly zero Poisson brackets. It does not break the symmetry of the model however. To clarify it we introduce the Dirac bracket[15], or equivalently "stared quantities"[20] which have weakly zero Poisson brackets with all the second class constraints. The modified super Poincare generators $({P^*}_\mu, {M^*}_{\mu\nu}, Q^{*j}) = (P_\mu, M_{\mu\nu}, Q^{*j})$ are also conserved and verifying the same super Poincare algebra. The modified supersymmetry transformations, generated by Q^{*j},

$$Q^{*j} = Q^j - \{Q^j, qr\}\frac{-1}{2(q^2 + r^2)}(q^2 - r^2), \tag{5.22}$$

are different from the original ones (4.4) and (4.6). Note there is ambiguity of higher power terms of constraints in the Hamiltonian supercharges in (5.22). In Appendix we show that there are corresponding supersymmetry transformations of the lagrangian (5.1) under which it is pseudo invariant *off-shell*.

6 Summary and Discussions

In this paper we have examined the rigid string model proposed in [7]. Especially we have discussed the massive sector of the model that have not been examined in [7]. It is quantized and the physical states are constructed explicitly using the representations of SO(3) and SU(1,1). The mass of the physical states are determined by their angular momenta and they are corresponding to the rotating rigid rods. The physical states are those with the highest angular momentum then lying on the leading trajectories. States with lower angular momenta, lying on daughter trajectories are unphysical. In a similar approach of the straight string model[8, 9] a different form of lagrangian was proposed starting from the NG type. The model is characterized by three first class constraints corresponding to three local gauge symmetries of the action. We can show that it is reduced to the present model when we impose two gauge fixing conditions appropriately .

We have also examined a possible supersymmetric model with the kappa symmetry. The L^{NG} action have target space supersymmetries but it requires an additional L^{WZ} action for the kappa symmetry. However the number of the independent kappa transformation is a half of the usual superstring cases. The kappa transformation parameters are restricted by two projection conditions (5.11) and (5.15).

In this model we can examine the BPS states correspondingly. The bosonic solution with $\theta_j = 0$ remains under combined super and kappa transformations if

$$\delta\theta_1 \;=\; \epsilon_1 + \frac{1-\Gamma_\kappa}{2}\, 2\not{P}\tilde{\kappa} = 0, \qquad \delta\theta_2 = \epsilon_2 - \frac{1-\Gamma_\kappa}{2}\, 2\not{P}\tilde{\kappa} = 0, \tag{6.1}$$

where $\frac{1-\Gamma_\kappa}{2}$ is the projection operators for the kappa transformations in (4.23). It gives BPS conditions that P_μ and r do not depend on τ. Since only a half of $\tilde{\kappa}$ is independent there remains 1/4 of the supersymmetry that preserves the BPS solutions in this model.

In section 4 we have started to construct the supersymmetric model by expecting that v_j^μ and r^μ are susy invariant and obtained only transformations invariant *on-shell*. We have shown that the *off-shell* invariant supersymmetry transformations are obtained by using the Dirac bracket or the modified supercharge Q^{*j}. In the corresponding lagrangian transformations neither v_j^μ nor r^μ are susy invariant. The forms of the *off-shell* transformations are not simple it is interesting to give any geometrical interpretation for example in superspace.

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A Appendix: Invariant Susy Transformations

We will show a sum of following four transformations 1∼4 of the lagrangian (5.1) becomes a total derivative then it is a symmetry transformation *off-shell*. Here we use the lagrangian variables

$$p^i_\mu \equiv p^i_\mu(x,v) = \frac{\partial L}{\partial \dot{x}^\mu_i}, \qquad q_\mu = \frac{1}{2}(p^1_\mu - p^2_\mu) \tag{A.1}$$

and they are satisfying the primary constraints (5.2) *identically*,

$$\frac{1}{2}(p_i^2 + (\frac{T}{2})^2 r^2) \equiv 0, \qquad p_i r \equiv 0. \tag{A.2}$$

On the other hand the secondary constraint

$$\chi = p_1 p_2 + 3(\frac{T}{2})^2 r^2 = -2(q^2 - (\frac{T}{2})^2 r^2) \equiv -2\,\hat{\chi} \tag{A.3}$$

does not vanish identically by $p^i_\mu \equiv \frac{\partial L}{\partial \dot{x}^\mu_i}$.

1: Original supersymmetry transformations $Q^j \epsilon_j$,

$$\begin{aligned} \delta_1\theta_j &= \epsilon_j, \quad \delta_1 x^\mu_i = -i\bar{\epsilon}_j B^{jk}\Gamma^\mu\theta_k, \quad \rightarrow \quad \delta_1 r^\mu = 0, \quad \delta_1 v^\mu_i = 0, && \text{(A.4)}\\ \delta_1 L^{tot} &= \delta_1 L^{WZ} = \partial_\tau(i\,r\,C^{jk}\bar{\epsilon}_j\Gamma_{11}\theta_k) - i\,\dot{r}\,C^{jk}\bar{\epsilon}_j\Gamma_{11}\theta_k. && \text{(A.5)} \end{aligned}$$

2: Transformation $F\hat{\chi}$,

$$\begin{aligned} \delta_2\theta^j &= 0, \quad \delta_2 x^\mu_1 = Fq^\mu, \quad \delta_2 x^\mu_2 = -Fq^\mu, && \text{(A.6)}\\ &\rightarrow \quad \delta_2 r^\mu = 2Fq^\mu, \quad \delta_2 v^\mu_1 = \partial_\tau(Fq^\mu), \quad \delta_2 v^\mu_2 = \partial_\tau(-Fq^\mu), \\ \delta_2 L^{tot} &= p^1_\mu\partial_\tau(Fq^\mu) + p^2_\mu\partial_\tau(-Fq^\mu) - (\mu_1 p^1_\mu + \mu_2 p^2_\mu)(2Fq^\mu) \\ &= \partial_\tau(F(q^2 + (\frac{T}{2})^2 r^2)) + \dot{F}\hat{\chi} - 2F\frac{\dot{r}}{r}\hat{\chi} - 4F(\frac{T}{2})^2 r\dot{r} && \text{(A.7)} \end{aligned}$$

3: Transformation $G\phi$,

$$\begin{aligned} \delta_3\theta^j &= 0, \quad \delta_3 x^\mu_1 = Gp^\mu_1, \quad \delta_3 x^\mu_2 = Gp^\mu_2, && \text{(A.8)}\\ &\rightarrow \quad \delta_3 r^\mu = 2Gq^\mu, \quad \delta_3 v^\mu_1 = \partial_\tau(Gp^\mu_1), \quad \delta_3 v^\mu_2 = \partial_\tau(Gp^\mu_2), \\ \delta_3 L^{tot} &= p^1_\mu\partial_\tau(Gp^\mu_1) + p^2_\mu\partial_\tau(Gp^\mu_2) - (\mu_1 p^1_\mu + \mu_2 p^2_\mu)(2Gq^\mu) \\ &= \partial_\tau(-2G(\frac{T}{2})^2 r^2) - 2G\frac{\dot{r}}{r}\hat{\chi}. && \text{(A.9)} \end{aligned}$$

4: Transformation $\zeta^j\hat{\rho}_j$,

$$\delta_4\theta_j = \hat{\rho}_j, \quad \delta_4 x^\mu_i = -iB^{jk}\overline{\theta}_j\Gamma^\mu\hat{\rho}_k, \quad \rightarrow \quad \delta_4 r^\mu = 0, \quad \delta_4 v^\mu_i = 2i\overline{\hat{\rho}}_j B^{jk}\Gamma^\mu\dot{\theta}_k, \tag{A.10}$$

$$\delta_4 L^{tot} = -2i\,\dot{\overline{\theta}}_j(B^{jk}\not{P} + C^{jk}_+\,r\,\Gamma_{11})\hat{\rho}_k - i\,\dot{r}\,C^{jk}\,\overline{\theta}_j\Gamma_{11}\hat{\rho}_k + \partial_\tau(ir\,C^{jk}\overline{\theta}_j\Gamma_{11}\hat{\rho}_k). \tag{A.11}$$

First we choose F so as to the last terms of (A.5) and (A.7) cancel,

$$F = \frac{-1}{4(\frac{T}{2})^2 r}(i\,\overline{\epsilon}_j C^{jk}\Gamma_{11}\theta_k), \quad \rightarrow \quad \dot{F} = \frac{-1}{4(\frac{T}{2})^2 r}(i\,\overline{\epsilon}_j C^{jk}\Gamma_{11}\dot{\theta}_k) - \frac{\dot{r}}{r}F. \tag{A.12}$$

Next a choice of $G = -\frac{3}{2}F$ makes sum of the first three transformations as

$$\delta_{123}L^{tot} \;=\; \partial_\tau\left(i\,r\,C^{jk}\overline{\epsilon}_j\Gamma_{11}\theta_k + F(q^2 + (\frac{T}{2})^2 r^2) - 2G(\frac{T}{2})^2 r^2\right) + \frac{-\hat{\chi}}{4(\frac{T}{2})^2 r}(i\,\overline{\epsilon}_j C^{jk}\Gamma_{11}\dot{\theta}_k). \tag{A.13}$$

Finally the last term of (A.13) can be cancelled with the first term of (A.11) by a choice of $\hat{\rho}_k$, using (4.20) and $k = \sqrt{2}T$,

$$\hat{\rho}_1 = -\hat{\rho}_2 = \frac{1}{4kr}(\not{P} - kr\Gamma_{11})\left((\beta+1)\epsilon_1 + (\beta-1)\epsilon_2\right), \tag{A.14}$$

where we have used $\hat{\chi} = -\frac{1}{4}(\not{P} - kr\Gamma_{11})^2$. Since the $\hat{\rho}_k$'s are verifying $C^{jk}\hat{\rho}_k = 0$ the second and third term of (A.11) vanish as well.

It completes a proof that the lagrangian (5.1) is invariant under the four combined transformations. The third and the forth transformations are essentially the diffeomorphism and the kappa transformations. The sum of first and second transformations is the modified supersymmetry transformation generated by Q^{*j} in the hamiltonian formalism given in (5.22).

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